Utopian preference mapping and the utopian preference method for group multiobjective optimization*

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Abstract The individual utopian preference and the group utopian preference on a set of alternatives, and the concept of the utopian preference mapping from the individual utopian preferences to the group utopian preferences, based on the utopian points of the corresponding multiobjective optimization models proposed by decision makers are introduced. Through studying the various fundamental properties of the utopian preference mapping, a method for solving group multiobjective optimization problems with multiple multiobjective optimization models is constructed.

Keywords: group decision making, multiobjective optimization, utopian point, utopian preference.

Group multiobjective optimization is a new interdisciplinary field of group decision making and multiobjective optimization. It combines quantitative and qualitative methods in the study of multiobjective optimization problems with multiple decision makers. Its theory and method has important applications in decision making processes in our modern society. From the 1980's, group multiobjective optimization problems based on one-multiobjective-optimization model shared by all decision makers with different individual preferences have received wide attention[1~3]. Later, multiobjective optimization problems based on multiple-multiobjective-optimization models proposed by different decision makers have also been studied[4~7]. The usual method for handling these two types of problems is to introduce appropriate utility functions, thereby transforming the problem into a single objective optimization problem, and at the same time aggregating individual preferences into a group preference. Because this traditional method usually transforms a group multiobjective problem into a usual numerical optimization problem, it in general can only obtain one optimal solution to a problem in some particular sense with regard to the group of decision makers.

For a group multiobjective optimization problem with different multiobjective optimization models, this paper defines the concepts of individual utopian preference and group utopian preference using the distance between the objective point of alternative and the utopian point of the multiobjective optimization problem given by decision makers. After discussing the fundamental properties of the utopian preference mapping from individual utopian preferences to group utopian preference, we construct a group sequencing method for all alternatives in group multiobjective optimization problems.

1 Model and utopian preference

Let $G=\{DM_1,\cdots,DM_l\}$ ($l\geqslant 2$) be a group of decision makers, and DM_r ($r=1,\cdots,l$) be the rth decision maker. Now consider the group multiobjective optimization problem:

$$G-\{V-\min_{x\in X}f^1(x),\cdots,V-\min_{x\in X}f^l(x)\},\qquad \text{(GVP)}$$

where $X=\{x^1,\cdots,x^s\}$ ($s\geqslant 3$) is a set of alternatives, and

$$V-\min_{x\in X}f^r(x)\quad (r=1,\cdots,l)\qquad (\mathrm{VP})_r$$

is a multiobjective optimization model given by DM_r, and $f^r: X\to R^{m_r}$ is the corresponding vector objective function. Let $f^r(x)=(f_1^r(x),\cdots,f_{m_r}^r(x))^{\mathrm{T}}$ ($r=1,\cdots,l$) and denote

$$\bar{f}_k^r=\min_{x\in X}f_k^r(x),\quad k=1,\cdots,m_r;\ r=1,\cdots,l.$$

Then

$$\bar{f}^r(x)=(\bar{f}_1^r,\cdots,\bar{f}_{m_r}^r)^{\mathrm{T}},\quad r=1,\cdots,l$$

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is the utopian point of the multiobjective optimization problem $(VP)_r$.

Denote the norm of Euclidean space R^{m_r} as $\|\cdot\|_{m_r}(r=1,\cdots,l)$.

Definition 1. Let R_r, P_r and $I_r(r=1,\cdots,l)$ be binary relations on the set X[8], and $\bar{f}^r$ be a utopian point of $(VP)_r$. For any x^i, $x^j\in X$, define

(i) $x^iR_rx^j\Leftrightarrow\|f^r(x^i)-\bar{f}^r\|_{m_r}\leqslant\|f^r(x^j)-\bar{f}^r\|_{m_r}$,

(ii) $x^iP_rx^j\Leftrightarrow\|f^r(x^i)-\bar{f}^r\|_{m_r}<\|f^r(x^j)-\bar{f}^r\|_{m_r}$,

(iii) $x^iI_rx^j\Leftrightarrow\|f^r(x^i)-\bar{f}^r\|_{m_r}=\|f^r(x^j)-\bar{f}^r\|_{m_r}$.

Then we call R_r, P_r and I_r utopian preference, strict utopian preference and utopian indifference of DM_r on X respectively.

Theorem 1. Suppose R_r is a utopian preference of $DM_r(r=1,\cdots,l)$ on X.

(i) For any x^i, x^j, $x^k\in X$, if $x^iR_rx^j$, $x^jR_rx^k$, then $x^iR_rx^k$.

(ii) For any x^i, $x^j\in X$, $x^iR_rx^j$ or $x^jR_rx^i$ holds, or both hold.

Proof. (i) Since $x^iR_rx^j$, $x^jR_rx^k$, by Definition 1(i), we have
$$\|f^r(x^i)-\bar{f}^r\|_{m_r}\leqslant\|f^r(x^j)-\bar{f}^r\|_{m_r}$$
and
$$\|f^r(x^j)-\bar{f}^r\|_{m_r}\leqslant\|f^r(x^k)-\bar{f}^r\|_{m_r},$$
hence $\|f^r(x^i)-\bar{f}^r\|_{m_r}\leqslant\|f^r(x^k)-\bar{f}^r\|_{m_r}$, that is, $x^iR_rx^k$.

(ii) Immediately from Definition 1(i).

Definition 2. Let $\lambda_r\geqslant 0(r=1,\cdots,l)$, $\sum_{i=1}^{l}\lambda_i=1$, $x\in X$, and $\bar{f}^r$ be a utopian point of $(VP)_r$. Then we call
$$D(x)=\sum_{r=1}^{l}\lambda_r\|f^r(x)-\bar{f}^r\|_{m_r}$$
the utopian distance of G at point x. Let R, P and I be binary relations on the set X, and for any x^i, $x^j\in X$, define

(i) x^iRx^j if and only if $D(x^i)\leqslant D(x^j)$,

(ii) x^iPx^j if and only if $D(x^i)<D(x^j)$,

(iii) x^iIx^j if and only if $D(x^i)=D(x^j)$.

Then we call R, P and I utopian preference, strict utopian preference and utopian indifference of G on X, respectively, and R and (P, I) correspond to each other.

Definition 3. Let $R_r(r=1,\cdots,l)$ and R be utopian preferences of DM_r and G on X, respectively, and let S be a subset of X.

(i) $U_r(S)=\{\bar{x}\in S\mid\bar{x}R_rx,\ \forall x\in S\}(r=1,\cdots,l)$ is called the utopian optimal set of DM_r on S, and $\bar{x}\in U_r(S)$ is called utopian optimal solution of DM_r on S.

(ii) $U(S)=\{\bar{x}\in S\mid\bar{x}Rx,\ \forall x\in S\}$ is called the utopian optimal set of G on S, and $\bar{x}\in U(S)$ is called utopian optimal solution of G on S. Specially, we call $\bar{x}\in U(X)$ the utopian optimal solution of the problem (GVP).

Theorem 2. Suppose the set $S\subset X$.

(i) $\bigcap_{r=1}^{l}U_r(S)\subset U(S)$.

(ii) If $\bigcap_{r=1}^{l}U_r(S)\neq\varnothing$, then $U(S)=\bigcap_{r=1}^{l}U_r(S)$.

Proof. (i) For any $\bar{x}\in\bigcap_{r=1}^{l}U_r(S)$, by Definition 3(i) and 1(i), we have
$$\|f^r(\bar{x})-\bar{f}\|_{m_r}\leqslant\|f^r(x)-\bar{f}^r\|_{m_r},$$
$$\forall x\in S,\ r=1,\cdots,l.$$
According to Definition 2, we have
$$D(\bar{x})=\sum_{r=1}^{l}\lambda_r\|f^r(\bar{x})-\bar{f}^r\|_{m_r}$$
$$\leqslant\sum_{r=1}^{l}\lambda_r\|f^r(x)-\bar{f}^r\|_{m_r}=D(x),$$
$$\forall x\in S,$$
that is,
$$\bar{x}Rx,\quad\forall x\in S.$$
Therefore, $\bar{x}\in U(S)$ by Definition 3(ii).

(ii) It is sufficient to show that $U(S)\subset\bigcap_{r=1}^{l}U_r(S)$. Now to the contrary we assume that there exists some $\bar{x}\in U(S)$ and $\bar{x}\notin\bigcap_{r=1}^{l}U_r(S)$, then by Definition 3(ii) and 2(i), we have

$$D(\bar{x}) \leqslant D(x), \quad \forall x \in S. \qquad (1)$$

And by the assumption we have some $\bar{x} \in \bigcap_{r=1}^{l} U_r(S) \neq \varnothing$, and $\bar{x} \neq \tilde{x}$, that is,

$$\| f^r(\bar{x}) - \tilde{f} \|_{m_r} \leqslant \| f^r(x) - \tilde{f}^r \|_{m_r},$$
$$\forall x \in S, \ r = 1, \cdots, l$$

from Definition 3(i) and 1(i). Specially, for $\tilde{x} \in U(S) \subset S$, we have

$$\| f^r(\bar{x}) - \tilde{f} \|_{m_r} \leqslant \| f^r(\tilde{x}) - \tilde{f}^r \|_{m_r},$$
$$r = 1, \cdots, l. \qquad (2)$$

Now since $\tilde{x} \notin \bigcap_{r=1}^{l} U_r(S)$, there exists some $\bar{r} \in \{1, \cdots, l\}$ such that $\tilde{x} \notin U_{\bar{r}}(S)$, but $\bar{x} \in U_{\bar{r}}(S)$ and $\bar{x} \neq \tilde{x}$, hence $\bar{x} P_{\bar{r}} \tilde{x}$. By Definition 1(ii), we have

$$\| f^{\bar{r}}(\bar{x}) - \tilde{f}^{\bar{r}} \|_{m_{\bar{r}}} < \| f^{\bar{r}}(\tilde{x}) - \tilde{f}^{\bar{r}} \|_{m_{\bar{r}}}.$$

Thus, by Eq. (2) and Definition 2, we obtain

$$D(\bar{x}) < D(\tilde{x}),$$

which contradicts (1).

2 Utopian preference mapping

In this section, we introduce a mapping from individual utopian preferences of decision makers to the group utopian preference.

Definition 4. Let R_r be the utopian preference of $DM_r (r = 1, \cdots, l)$ on X, and R be the utopian preference of G on X, the mapping $U: \{R_1, \cdots, R_l\} \to R$ is called the utopian preference mapping on X, and is denoted as $R = U(R_1, \cdots, R_l)$.

Definition 5. Let f^r be the vector objective function of $(\mathrm{VP})_r$ proposed by $DM_r (r = 1, \cdots, l)$. Then the vector objective functions $\{f^1, \cdots, f^l\}$ is called a multiobjective intercepting surface of G on X, and is denoted as $[f^1, \cdots, f^l]_X$. And the utopian preferences $\{R_1, \cdots, R_l\}$ determined by $f^r (r = 1, \cdots, l)$ under Definition 1(i) is called a utopian preference intercepting surface corresponding to $[f^1, \cdots, f^l]_X$ of G on X, and is denoted as $[R_1, \cdots, R_l]_X$.

Following are some fundamental properties of the utopian preference mapping.

Theorem 3. Let $R = U(R_1, \cdots, R_l)$.

(i) For any $x^i, x^j, x^k \in X$, if $x^i R x^j$, $x^j R x^k$, then $x^i R x^k$.

(ii) For any $x^i, x^j \in X$, $x^i R x^j$ or $x^j R x^i$ holds, or both hold.

Proof. (i) By Definition 2(i), $x^i R x^j$ and $x^j R x^k$ imply

$$D(x^i) \leqslant D(x^j) \leqslant D(x^k),$$

which means $x^i R x^k$.

(ii) Immediately from Definition 2.

Now suppose $[f^1, \cdots, f^l]_X$ and $[f^{1'}, \cdots, f^{l'}]_X$ are two multiobjective intercepting surfaces of G on X, and $[R_1, \cdots, R_l]_X$ and $[R_1', \cdots, R_l']_X$ are the corresponding utopian preference intercepting surfaces.

Theorem 4. Let $R = U(R_1, \cdots, R_l)$, $R' = U(R_1', \cdots, R_l')$, the set $S \subset X$, and denote $U_r'(S) = \{\tilde{x} \in S \mid \tilde{x} R_r' x, \forall x \in S\} (r = 1, \cdots, l)$ and $U'(S) = \{\tilde{x} \in S \mid \tilde{x} R' x, \forall x \in S\}$. If

$$U_r(S) = U_r'(S), \quad r = 1, \cdots, l,$$

then $U(S) = U'(S)$.

Proof. For any $\tilde{x} \in S$, since $U_r(S) = U_r'(S)$ $(r = 1, \cdots, l)$, by Definition 3 (i), we have

$$\tilde{x} R_r x \Leftrightarrow \tilde{x} R_r' x, \quad \forall x \in S, \ r = 1, \cdots, l.$$

According to Definition 1(i), that is

$$\| f^r(\tilde{x}) - \tilde{f}^r \|_{m_r} \leqslant \| f^r(x) - \tilde{f}^r \|_{m_r} \Leftrightarrow \| f^{r'}(\tilde{x}) - \tilde{f}^{r'} \|_{m_r} \leqslant \| f^{r'}(x) - \tilde{f}^{r'} \|_{m_r},$$
$$\forall x \in S, \ r = 1, \cdots, l,$$

therefore

$$D(\tilde{x}) = \sum_{r=1}^{l} \lambda_r \| f^r(\tilde{x}) - \tilde{f}^r \|_{m_r} \leqslant \sum_{r=1}^{l} \lambda_r \| f^r(x) - \tilde{f}^r \|_{m_r} = D(x)$$
$$\Leftrightarrow D'(\tilde{x}) = \sum_{r=1}^{l} \lambda_r \| f^{r'}(\tilde{x}) - \tilde{f}^{r'} \|_{m_r} \leqslant \sum_{r=1}^{l} \lambda_r \| f^{r'}(x) - \tilde{f}^{r'} \|_{m_r} = D'(x),$$
$$\forall x \in S.$$

Thus, by Definition 2 (i), we have

$$\tilde{x} R x \Leftrightarrow \tilde{x} R' x, \quad \forall x \in S,$$

hence we have $U(S) = U'(S)$.

Theorem 5. For any $x^i, x^j \in X (i, j = 1, \cdots, s)$, there exists a multiobjective intercepting surface $[\bar{f}^1, \cdots, \bar{f}^l]_X$ of G on X, such that the corresponding utopian preference intercepting surface $[\bar{R}_1, \cdots, \bar{R}_l]_X$ generates a preference $\bar{R} = U(\bar{R}_1, \cdots, \bar{R}_l)$ satisfying

$$x^i\bar{P}x^j,$$

where $\bar{P}$ corresponds to $\bar{R}$.

Proof. For any x^i, $x^j \in X$, we choose $[\bar{f}^1, \cdots, \bar{f}^l]_X$ such that

$$\| \bar{f}^r(x^i) - \tilde{\bar{f}}^r \|_{m_r} \leqslant \| \bar{f}^r(x) - \tilde{\bar{f}}^r \|_{m_r}, \quad r = 1, \cdots, l,$$

and at least one of them is "<", then we have

$$\bar{D}(x^i) = \sum_{r=1}^{l} \lambda_r \| \bar{f}^r(\tilde{x}) - \tilde{\bar{f}}^r \|_{m_r} < \sum_{r=1}^{l} \lambda_r \| \bar{f}^r(x) - \tilde{\bar{f}}^r \|_{m_r} = \bar{D}(x^j).$$

Therefore, according to Definition 2 (ii), we obtain $x^i\bar{P}x^j$.

Theorem 6. Let $R = U(R_1, \cdots, R_l)$, and $P_r(r=1,\cdots,l)$ and P correspond to R_r and R respectively. Then there exists no $t \in \{1, 2, \cdots, l\}$, $\lambda_t \leqslant \sum_{r \neq t} \lambda_r$, such that

$$x^iP_tx^j \Rightarrow x^iPx^j, \quad \forall x^i, x^j \in X.$$

Proof. For any x^i, $x^j \in X$, equivalently we show that there exists $f^r(r=1,\cdots,l, r\neq t)$ such that $[R_1, \cdots, R_t, \cdots, R_l]_X$ corresponding to $[f^1, \cdots, f^t, \cdots, f^l]_X$ generates $R = U(R_1, \cdots, R_l)$ which satisfies x^jRx^i.

In fact, $x^iP_tx^j$ means, by Definition 1 (ii),

$$\| f^t(x^i) - \tilde{f}^t \|_{m_t} < \| f^t(x^j) - \tilde{f}^t \|_{m_t}. \quad (3)$$

Now we choose $f^r(r=1,\cdots,l, r\neq t)$ such that

$$\| f^r(x^i) - \tilde{f}^r \|_{m_r} - \| f^r(x^j) - \tilde{f}^r \|_{m_r} = \| f^t(x^j) - \tilde{f}^t \|_{m_t} - \| f^t(x^i) - \tilde{f}^t \|_{m_t} > 0.$$

Since $\lambda_t \leqslant \sum_{r\neq t} \lambda_r$ and (3) hold, we have

$$\begin{aligned} D(x^i) - D(x^j) &= \sum_{r=1}^{l} \lambda_r \| f^r(x^i) - \tilde{f}^r \|_{m_r} - \sum_{r=1}^{l} \lambda_r \| f^r(x^j) - \tilde{f}^r \|_{m_r} \\ &= \sum_{r\neq t} \lambda_r [\| f^r(x^i) - \tilde{f}^r \|_{m_r} - \| f^r(x^j) - \tilde{f}^r \|_{m_r}] \\ &\quad + \lambda_t [\| f^t(x^i) - \tilde{f}^t \|_{m_t} - \| f^t(x^j) - \tilde{f}^t \|_{m_t}] \\ &= \sum_{r\neq t} \lambda_r [\| f^t(x^j) - \tilde{f}^t \|_{m_t} - \| f^t(x^i) - \tilde{f}^t \|_{m_t}] \\ &\quad - \lambda_t [\| f^t(x^j) - \tilde{f}^t \|_{m_t} - \| f^t(x^i) - \tilde{f}^t \|_{m_t}] \\ &= \left(\sum_{r\neq t} \lambda_r - \lambda_t \right) [\| f^t(x^j) - \tilde{f}^t \|_{m_t} - \| f^t(x^i) - \tilde{f}^t \|_{m_t}] \geqslant 0, \end{aligned}$$

which means x^jRx^i by Definition 2(i).

From the above Theorem 3 ~ Theorem 6, we know that the utopian preference mapping $U: \{R_1, \cdots, R_l\} \to R$ satisfies the properties tantamount to Arrow's axioms[9], except the consistency axiom. The condition $\lambda_t \leqslant \sum_{r\neq t} \lambda_r$ in Theorem 6 means the weight of decision maker DM_t can not exceed the sum of other decision makers' weights. Otherwise, DM_t will become a dictator.

3 Utopian preference method

Since a utopian preference mapping possesses the main properties of reasonable group decision making, we would like to make use of it by giving out a preference sequencing method of alternatives for the group multiobjective optimization model (GVP).

(i) Calculating objective point. For $(\mathrm{VP})_r$ provided by $DM_r(r = 1, \cdots, l)$, calculate the objective values at all alternatives $x^i \in X(i=1,\cdots,s)$, $f_k^r(x^i)$ $(k=1,\cdots,m_r; r=1,\cdots,l)$ to obtain the corresponding objective points:

$$f^r(x^i) = (f_1^r(x^i), \cdots, f_{m_r}^r(x^i))^{\mathrm{T}}, \quad r = 1, \cdots, l.$$

(ii) Finding the utopian point. Find the utopian (optimal) value of each objective,

$$\tilde{f}_k^r = \min_{x\in X} f_k^r(k), \quad k = 1, \cdots, m_r; \ r = 1, \cdots, l,$$

to obtain the utopian point of each $(\mathrm{VP})_r$:

$$\tilde{f}^r = (\tilde{f}_1^r, \cdots, \tilde{f}_{m_r}^r)^{\mathrm{T}}, \quad r=1,\cdots,l.$$

(iii) Calculating the utopian distance. Determine the weight coefficient $\lambda_t \geqslant 0$ for each $DM_r(r=1,\cdots,l)$ and make sure $\sum_{r=1}^{l} \lambda_r = 1$, and for any $t \in \{1, \cdots, l\}$, $\lambda_t \leqslant \sum_{r\neq t} \lambda_r$. For some kind of norm $\| \cdot \|_{m_r}$ in R^{m_r}, calculate the utopian distance of G at x^i:

$$D(x^i) = \sum_{r=1}^{l} \lambda_r \| f^r(\tilde{x}) - \tilde{f}^r \|_{m_r}, \quad i = 1, \cdots, s.$$

(iv) The utopian preference sequencing. For $\tilde{x}^i \in X(i=1,\cdots,s)$, if

$$D(\tilde{x}^i) \leqslant D(\tilde{x}^{i+1}), \quad i = 1, \cdots, s-1,$$

then we obtain the utopian preference sequencing:

$$\tilde{x}^1R\tilde{x}^2R\cdots R\tilde{x}^s$$

for the alternatives.

Theorem 7. Suppose $[f^1, \cdots, f^l]_X$ is a multiobjective intercepting surface of G on X, and $[R_1, \cdots, R_l]_X$ is the corresponding utopian preference intercepting surface. Let $R = U(R_1, \cdots, R_l)$, and let P be the strict utopian preference corresponding to R.

(i) For x^i, $x^j \in X$, if $D(x^i) \leqslant D(x^j)$, then $x^i R x^j$.

(ii) For x^i, $x^j \in X$, if $D(x^i) < D(x^j)$, then $x^i P x^j$.

(iii) For $\tilde{x} \in X$, if $D(\tilde{x}) \leqslant D(x)(\forall x \in X)$, then $\tilde{x} \in U(X)$.

Proof. (i) Immediately from Definition 2 (i).

(ii) Immediately from Definition 2 (ii).

(iii) Since

$$D(\tilde{x}) \leqslant D(x), \quad \forall x \in X,$$

by Definition 2 (i), we have

$$\tilde{x} R x, \quad \forall x \in X,$$

which results in $\tilde{x} \in U(X)$ immediately from $\tilde{x} \in X$ and Definition 3 (ii).

From this theorem, we know, for a group multiobjective optimization problem (GVP), according to the utopian preference method, a group preference sequencing can be constructed for all alternatives by the utopian distance. Specially, $\tilde{x}^1$ is just the utopian optimal solution of (GVP).

Example. Let $G = \{DM_1, DM_2, DM_3\}$ be a decision maker group. Now we consider a group multiobjective optimization problem:

$$G - \{V - \min_{x \in X} f^1(x), V - \min_{x \in X} f^2(x), V - \min_{x \in X} f^3(x)\}, \tag{4}$$

where the set of alternatives $X = \{x^1 = 2, x^2 = 1, x^3 = 3\}$, and the vector objective function for the 3 decision makers is $f^1(x) = (2x, 1 - x, x^2)^{\mathrm{T}}$, $f^2(x) = (x, 2x - 1)^{\mathrm{T}}$ and $f^3(x) = (-x, x + 1, 1 - x^2)^{\mathrm{T}}$. Now we give out the preference sequencing of alternatives for the group multiobjective optimization problem by utopian preference method as follows:

(i) Calculating objective point. Obviously we have

$$f^1(x^1) = (4, -1, 4)^{\mathrm{T}}, \quad f^2(x^1) = (2, 3)^{\mathrm{T}},$$
$$f^3(x^1) = (-2, 3, -3)^{\mathrm{T}};$$
$$f^1(x^2) = (2, 0, 1)^{\mathrm{T}}, \quad f^2(x^2) = (1, 1)^{\mathrm{T}},$$
$$f^3(x^2) = (-1, 2, 0)^{\mathrm{T}};$$
$$f^1(x^3) = (6, -2, 9)^{\mathrm{T}}, \quad f^2(x^3) = (3, 5)^{\mathrm{T}},$$
$$f^3(x^3) = (-3, 4, -8)^{\mathrm{T}}. \tag{5}$$

(ii) Finding the utopian point. From (5) we can find the utopian (optimal) value of each objective is

$$\tilde{f}^1 = (2, -2, 1)^{\mathrm{T}}, \quad \tilde{f}^2 = (1, 1)^{\mathrm{T}},$$
$$\tilde{f}^3 = (-3, 2, -8)^{\mathrm{T}}.$$

(iii) Calculating the utopian distance. Suppose the weight coefficient $\lambda_r = 1/3$ for each $DM_r (r = 1, 2, 3)$, and for the normal norm $\| \cdot \|_m$ in R^m, $\| y \|_m = \sqrt{y_1^2 + \cdots + y_m^2}$, for $y = (y_1, \cdots, y_m)^{\mathrm{T}}$, we can calculate the utopian distance of G at $x^i (i = 1, 2, 3)$ as

$$D(x^1) = \frac{1}{3}[\sqrt{14} + \sqrt{5} + \sqrt{27}] = 3.7247,$$
$$D(x^2) = \frac{1}{3}[\sqrt{4} + 0 + \sqrt{68}] = 3.4154,$$
$$D(x^3) = \frac{1}{3}[\sqrt{16} + \sqrt{20} + \sqrt{4}] = 3.4907.$$

(iv) The utopian preference sequencing. Since $D(x^2) < D(x^3) < D(x^1)$, we have utopian preference sequencing $x^2 R x^3 R x^1$ (in fact $x^2 P x^3 P x^1$).

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